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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2015
SECOND YEAR [BATCH 2014-17]
MATH FOR ECO [General]
Paper : III

Date : 23/12/2015
Time : 11 am – 2 pm
Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

(*Answer **any five** questions*)

1. a) Let $f(x,y)=\begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} &, (x,y)\neq(0,0) \\ 0 &, (x,y)=(0,0) \end{cases}$

   check whether f is continuous and differentiable at (0,0)? [7]

   b) State Schwarz's theorem for equality of mixed derivatives. Show that the conditions are not necessary. [3]

2. a) State and prove Euler's theorem for a homogeneous function of three variables. [5]

   b) If $V=\cos^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, then verify cos V is a homogeneous function of x, y of degree $\dfrac{1}{2}$. Hence prove that $x\dfrac{\partial V}{\partial x}+y\dfrac{\partial V}{\partial y}+\dfrac{1}{2}\cot V=0$. [5]

3. a) The roots of the equation in λ, $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ are u, v, w. Prove that the Jacobian $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}=-2\dfrac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$. [5]

   b) Show that the expression $ax^2+by^2+cz^2+2fyz+2gzx+2hxy$ can be resolved into linear factors if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$. [5]

4. a) Prove that the function $f(x,y)=|x|+|y|$ has a minimum at (0,0). Is there any maximum? [5]

   b) Determine relative extreme values and Saddle points for the function $f(x,y)=x^6+(x-y)^3$, $(x,y)\in\mathbb{R}^2$. [5]

5. a) Define a convex function and find the range of values of x for which $y=x^4-6x^3+12x^2+5x+7$ is convex or concave. Find also its point of inflexion, if any. [5]

   b) Use the method of Lagrange multiplier to minimize the function $f(x,y)=x^2-8x+y^2-12y+48$ subject to the constraint x + y = 8. Assume x, y are all nonnegative. [5]

6. a) Evaluate $\lim\limits_{x\to 0}\dfrac{\int_0^{x^2} e^{\sqrt{1+t}}\,dt}{x^2}$. [5]

   b) Show that continuity of f is not a necessary condition for the existence of an antiderivative of f. [5]

7. a) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ is convergent iff m, n > 0. [5]

b) Prove that the integral $\int_0^\infty x^{m-1}e^{-x}dx$ is convergent iff m > 0. [5]

8. a) $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$. Show that $\frac{\partial x}{\partial r}\cdot\frac{\partial r}{\partial x}+\frac{\partial x}{\partial \theta}\cdot\frac{\partial \theta}{\partial x}+\frac{\partial x}{\partial \phi}\cdot\frac{\partial \phi}{\partial x}=1$. [5]

b) Define the Directional derivative of f(x,y) at (x_0, y_0) along a unit vector $\vec{n}=(n_1,n_2)$. [2]

c) Find the derivative of f(x, y) = $x^2 + y^2$ in the direction of (1, 2) at the point (1, 1, 2). [3]

Group – B

(*Answer* ***any five*** *questions*)

9. a) State the existence and uniqueness theorem of first order ordinary differential equation. [2]

b) Find the order and degree of $\left(\frac{d^2y}{dx^2}\right)^3+\frac{dy}{dx}=e^x$. [3]

10. Check whether $(xy^2+x)dx+x^2ydy=0$ is an exact differential equation? [5]

11. a) Define first order homogeneous differential equation. [2]

b) Check whether $(x+2y)dx+xdy=0$ is a homogeneous differential equation? [3]

12. Show that the differential equation satisfied by the family of curves given by $c^2+2cy-x^2+1=0$ where c is the parameter of the family, is $(1-x^2)p^2+2xyp+x^2=0$, where $p\equiv\frac{dy}{dx}$. [5]

13. Obtain the complete primitive of the equation $y=px+\sqrt{1+p^2}$ and also show that the singular solution is the circle which passes through the origin. [5]

14. Solve : $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$. [5]

15. Solve : $ayp^2+(2x-b)p-y=0$, a > 0 and $p\equiv\frac{dy}{dx}$. [5]

16. a) Solve : $xdx+ydy=a(xdy-ydx)$, a is constant. [2]

b) Solve : $(1+xy)ydx+x(1-xy)dy=0$. [3]

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